Worksheet #11: Implicit Differentiation

Generalized Derivative Formulas:

$\frac{d}{dx}[\sin(u)]=\cos(u)\frac{du}{dx}$ $\quad$ $\frac{d}{dx}[\cos(u)]=-\sin(u)\frac{du}{dx}$

$\frac{d}{dx}[\tan(u)]=\sec^2(u)\frac{du}{dx}$ $\quad$ $\frac{d}{dx}[\cot(u)]=-\csc^2(u)\frac{du}{dx}$

$\frac{d}{dx}[\sec(u)]=\sec(u)\tan(u)\frac{du}{dx}$ $\quad$ $\frac{d}{dx}[\csc(u)]=-\csc(u)\cot(u)\frac{du}{dx}$

Power Rule: $\frac{d}{dx}[(f(x))^n]=n(f(x))^{n-1}f'(x)$ or $\frac{d}{dx}[(u(x))^n]=n[(u(x))^{n-1}]\frac{du}{dx}=\frac{d}{dx}[u^n]=nu^{n-1}\frac{du}{dx}$

Exponent Rule: $\frac{d}{dx}e^{f(x)}=e^{f(x)}f'(x)$ or $\frac{d}{dx}\left[e^{u(x)}\right]=e^{u(x)}\frac{du}{dx}$

Chain Rule: $[f(g(x))]'=f'(g(x))g'(x)$ or $\frac{d}{dx}[y(u)]=\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$

Product Rule: $\frac{d}{dx}[f(x)g(x)]=f(x)\frac{d}{dx}[g(x)]+g(x)\frac{d}{dx}[f(x)]=fg'+gf';\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}=uv'+vu'$

Quotient Rule: $\frac{d}{dx}[\frac{f(x)}{g(x)}]=\frac{g(x)\frac{d}{dx}[f(x)]-f(x)\frac{d}{dx}[g(x)]}{[g(x)]^2}=\frac{gf'-fg'}{g^2}$

1. Find $\frac{dy}{dx}$ for the following if $y=y(x)$:

   a. $xy^2=\tan(3x)$

   b. $3y^2+\sec(y)=4x^5+9$

   c. $5x^2+\csc(2xy)=\sin(y)$

   d. $\cos^2(y+x)=4x^5+9$

2. Find $\frac{d^2y}{dx^2}$ for the following if $y=y(x)$:

   a. $2y^2-3x^2=8$

   b. $x^3y^2=8$

   c. $e^{x^2}=\sin y$