Worksheet #17: Hyperbolic Trigonometric Functions

Useful Formulas:

$\cosh^2(x) - \sinh^2(x) = 1$

Hyperbolic Sine: $\sinh(x) = \frac{e^x - e^{-x}}{2}$

Hyperbolic Cosine: $\cosh(x) = \frac{e^x + e^{-x}}{2}$

$\tanh(u) = \frac{\sinh(x)}{\cosh(x)}$, $\coth(u) = \frac{\cosh(x)}{\sinh h(x)}$, $\sec h(u) = \frac{1}{\cosh(x)}$, $\csc h(u) = \frac{1}{\sinh(x)}$

$\frac{d}{dx}[\sinh(x)] = \cosh(x)$

$\frac{d}{dx}[\cosh(x)] = \sinh(x)$

$\frac{d}{dx}[\tanh(x)] = \mathrm{sech}^2(x)$

$\frac{d}{dx}[\coth(x)] = -\mathrm{csch}^2(x)\frac{du}{dx}$

$\frac{d}{dx}[\mathrm{sech}(x)] = -\mathrm{sech}(x)\tanh(x)$

$\frac{d}{dx}[\mathrm{csch}(x)] = -\mathrm{csch}(x)\coth(x)$

1. Starting from the definition of $\cosh(x)$ in terms of exponentials, show that its derivative is $\sinh(x)$.

2. Find the derivative of each function:

a) $y = \sinh^5(\sqrt{x^2 + 1})$

b) $y = \tan^{-1}(\sqrt{x})\tanh(\sqrt{x})$

3. Find the exact value of $f''(0)$ and $f''(\ln 2)$ for the function $f(x) = \cosh(2x)$.

4. Start with the equation $y = \cosh^{-1}(x)$ with $x > 1$:

a) Convert to an equation with the regular (not inverse) hyperbolic cosine

b) Use the definition of hyperbolic cosine in terms of exponentials to rewrite the equation from part a. Then convert it to a quadratic equation in e^y.

c) Choose the root of the quadratic that will make $y > 0$ (that is $e^y > 1$) to get $y = \cosh^{-1}(x) = \ln(x + \sqrt{x^2 - 1})$

d) Use the result from part c to show $\frac{d}{dx}\left[\cosh^{-1}(x)\right] = \frac{1}{\sqrt{x^2 - 1}}$.