

Worksheet #19: Rolle's Theorem and MVT

Basic Guidelines: For either Rolle's Theorem or the Mean Value Theorem to apply, the function $f(x)$ must be continuous on a closed interval $[a, b]$ and differentiable on the open interval (a, b) .

Rolle's Theorem:

If $f(a) = 0$ and $f(b) = 0$ then there is at least one value c with $a < c < b$ where $f'(c) = 0$.

Mean Value Theorem:

There is at least one value c with $a < c < b$ where $f'(c) = \frac{f(b) - f(a)}{b - a}$.

Find the value c that satisfies the conclusion of Rolle's Theorem for the functions on the given intervals:

1. $f(x) = x^2 - 10x$ on $[0, 10]$

2. $f(x) = x^3 - 3x$ on $[0, \sqrt{3}]$

3. $f(x) = \cos x + \sin x - 1$ on $[0, \frac{\pi}{2}]$

Find the value c that satisfies the Mean Value Theorem for the functions on the given intervals:

4. $f(x) = x^2 + 3x + 1$ on $[3, 5]$

5. $f(x) = x^3 + 1$ on $[0,3]$

6. A road runs in a straight line past a police station. A police car sitting 5 miles down the road from the station clocks a car traveling at 50 mph at 8:00AM. At 8:06AM, another police car sitting 15 miles from the police station clocks the same car travelling at 55 mph. If the speed limit on the road is 60 mph, can the police conclude that the car was speeding?

HINT: Use the Mean Value Theorem and the fact that the car's instantaneous velocity (what the speedometer reads) is given by the derivative.