Worksheet #2: Limit Theorems, Computing Limits

Definition: For $c \in \Re$, if $\lim_{x \to c} f(x) = L_1$, and $\lim_{x \to c} g(x) = L_2$, then

$$\lim_{x \to c}[f(x) \pm g(x)] = \lim_{x \to c} f(x) \pm \lim_{x \to c} g(x) = L_1 \pm L_2$$

$$\lim_{x \to c}[f(x) \cdot g(x)] = [\lim_{x \to c} f(x)] \cdot [\lim_{x \to c} g(x)] = L_1 \cdot L_2$$

$$\lim_{x \to c}[\frac{f(x)}{g(x)}] = \frac{\lim_{x \to c} f(x)}{\lim_{x \to c} g(x)} = \frac{L_1}{L_2}, \text{ if } L_2 \neq 0$$

$$\lim_{x \to c} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \to c} f(x)} = \sqrt[n]{L_1}$$, if n is even, we assume $L_1 \geq 0$.

Some helpful hints in computing the limits:

Step 1: Try to evaluate by substituting in the number (this works if the function is "continuous" at the c-value).

Step 2: If $\frac{0}{0}$ is obtained from step 1, try to simplify using algebra: factor, multiply by the conjugate, etc…

Find the limits for the following:

1. $\lim_{y \to 2} \frac{(y-1)(y-2)}{y+1}$

2. $\lim_{x \to 4} \frac{x^2 - 16}{x - 4}$

3. $\lim_{y \to 4} \frac{4 - y}{2 - \sqrt{y}}$

4. $\lim_{x \to 1} \frac{x^4 - 1}{x - 1}$

5. $\lim_{x \to 0} \frac{\sqrt{3x + 16} - 4}{x}$