Worksheet #20: L'Hôpital's Rule; Indeterminate Forms

Guidelines for using L'Hôpital's Rule:

1: $\dfrac{0}{0}, \dfrac{\infty}{\infty} \Rightarrow$ Apply L'Hôpital's Rule.   2: $0 \cdot \infty \Rightarrow$ Invert one term to the denominator.

3: $1^{\infty}, 0^{0}, \infty^{0} \Rightarrow$ Use *ln*, then take the limit.   4: $\infty - \infty \Rightarrow$ Find a common denominator or multiply by the conjugate, etc.

L'Hôpital's Rule: If $\lim\limits_{x \to a} \dfrac{f(x)}{g(x)}$ is in the form $\dfrac{0}{0}$ or $\dfrac{\infty}{\infty}$, then $\lim\limits_{x \to a} \dfrac{f(x)}{g(x)} = \lim\limits_{x \to a} \dfrac{f'(x)}{g'(x)}$ if it exists.

Find the limit for the following:

1. a) $\lim\limits_{x \to 1} \dfrac{\ln(x)}{x-1}$

   b) $\lim\limits_{x \to 0} \dfrac{\sin^{-1}(3x)}{x^3}$

2. a) $\lim\limits_{x \to 0} \dfrac{\sin(3x)}{x}$

   b) $\lim\limits_{x \to 0} \dfrac{x^2 + \cos x - 1}{2x^3 + 4x^2}$

3. a) $\lim\limits_{x \to +\infty} \dfrac{\ln(x)}{x}$

   b) $\lim\limits_{x \to +\infty} x^2 e^{-5x}$