Worksheet #22B: Derivatives & Graphs II

Basic Guidelines: Intervals on which $f(x)$ is **Increasing, Decreasing, Concave Up** or **Concave Down** and finding **Relative Extrema** by the **First Derivative Test and the Second Derivative Test.**

* To find the intervals where a function is **increasing** ($f'(x) \geq 0$) or **decreasing** ($f'(x) \leq 0$):
  Take the first derivative, find the critical points and then make a sign analysis.
* To find the intervals where a function is **concave up** ($f''(x) > 0$) or **concave down** ($f''(x) < 0$):
  Take the second derivative and make a sign analysis.
* To find the *x*-coordinates of all **inflection points**: *Look for a change of concavity where* $f''(x)$ *changes sign.*

Finding **Relative Extrema** with the **First Derivative Test**: *Let c be a critical point of f.*
* *For* $x < c, f'(x) < 0,$ *f(x) is decreasing, and for* $x > c, f'(x) > 0,$ *f(x) is increasing* $\Rightarrow$ *f(c) is a Relative Minimum.* $f'(x)$ *changes sign about x=c.*
* *For* $x < c, f'(x) > 0,$ *f(x) is increasing, and for* $x > c, f'(x) < 0,$ *f(x) is decreasing* $\Rightarrow$ *f(c) is a Relative Maximum.* $f'(x)$ *changes sign about x=c.*

Finding **Relative Extrema** with the **Second Derivative Test**: *Let* $f'(c) = 0$*, c is a stationary point.*
* *Substitute the critical point c into the second derivative to test the concavity.*
* $f''(c) < 0 \Rightarrow$ *f(c) is a Relative Maximum since f(x) is concave down about x=c.*
* $f''(c) > 0 \Rightarrow$ *f(c) is a Relative Minimum since f(x) is concave up about x=c.*

1. Utilize the Second Derivative Test to find the maximum and minimum values of the following function:

$$f(x) = x^3 - 12x + 5$$

a) First, identify the critical points of $f(x)$.

b) Now, plug the critical points into the second derivative $f''(x)$ and interpret the result.

c) Now, graph the function and verify your results.

2. Show that the Second Derivative Test fails to provide enough information to identify all the maximum and minimum values of the following function:

$$f(x)=4x^5-5x^4+2$$

Utilize the First Derivative Test to complete the problem and find the remaining maxs or mins. Then graph the function and verify your results.

Hint: At $x=\frac{3}{4}$, the corresponding point on the graph is approximately $(.75, 1.37)$.

3. Graph the function $f(x)=\sqrt[3]{x^2}$, identifying any maxs, mins, and points of inflection.

4. Given $f(x)=x-\sqrt[3]{x}$, identifying any maxs, mins, and points of inflection.

Hint: At $x = \pm\frac{1}{3\sqrt{3}}$, the corresponding points on the graph are approximately $(.19, -0.38)$ and $(-.19, 0.38)$.