

**Worksheet #23:** Graphing (Rational Functions, Vertical Asymptotes, Horizontal Asymptotes and Cusps)

**Basic Guidelines:**

- a) Find any vertical asymptotes and horizontal asymptotes.
- b) Find all critical points and make a sign analysis of  $f'(x)$ .
- c) Find any intervals where  $f(x)$  is increasing or decreasing.
- d) Find any relative extrema by using the First Derivative Test or Second Derivative Test.
- e) The graph will have a horizontal tangent line anywhere that  $f'(x) = 0$ .
- f) The graph will have a vertical tangent line anywhere that  $f'(x)$  is undefined but NOT a vertical asymptote.
- g) Make a sign analysis of  $f''(x)$ .
- h) Find any open intervals where  $f(x)$  is concave up or concave down.
- i) Find the  $x$ -coordinates of any inflection points.
- j) Make a sketch of the graph and label the coordinates where we have relative extrema and inflection points.

1. Graph the function:  $f(x) = \frac{x^2}{4 - x^2}$

2. Determine where the following functions have Vertical Tangent Lines or Horizontal Tangent Lines:  
(Hint: You calculated these derivatives already in Lab 19.)

$$f(x) = \sqrt[3]{x^2}$$

$$f(x) = x - \sqrt[3]{x}$$

3. Graph the function:  $f(x) = \frac{x}{x^2 - 2x + 1}$