Worksheet #27: Antiderivatives

The *Antiderivative* of a function $f(x)$ is another function $F(x)$ with the property that $F'(x) = f(x)$.

A general antiderivative formula ends in an arbitrary constant "+*C*".

In general, the antidifferentiation formulas are the reverse of corresponding differentiation formulas:

1. $(x)' = 1$	1. $f(x) = 1 \Rightarrow F(x) = x + C$
2. $(x^n)' = nx^{n-1}$	2. $f(x) = x^n \Rightarrow F(x) = \frac{1}{n+1}x^{n+1} + C$ (Note that $n \neq -1$)
3. $(\sin x)' = \cos x$	3. $f(x) = \cos x \Rightarrow F(x) = \sin x + C$
4. $(\cos x)' = -\sin x$	4. $f(x) = \sin x \Rightarrow F(x) = -\cos x + C$
5. $(\tan x)' = \sec^2 x$	5. $f(x) = \sec^2 x \Rightarrow F(x) = \tan x + C$
6. $(\sec x)' = \sec x \tan x$	6. $f(x) = \sec x \tan x \Rightarrow F(x) = \sec x + C$
7. $(\csc x)' = -\csc x \cot x$	7. $f(x) = \csc x \cot x \Rightarrow F(x) = -\csc x + C$
8. $(\cot x)' = -\csc^2 x$	8. $f(x) = \csc^2 x \Rightarrow F(x) = -\cot x + C$
9. $(\sin^{-1} x)' = \frac{1}{\sqrt{1-x^2}}$	9. $f(x) = \frac{1}{\sqrt{1-x^2}} \Rightarrow F(x) = \sin^{-1} x + C$
10. $(\tan^{-1} x)' = \frac{1}{1+x^2}$	10. $f(x) = \frac{1}{1+x^2} \Rightarrow F(x) = \tan^{-1} x + C$
11. $(\sec^{-1} x)' = \frac{1}{\lvert x\rvert\sqrt{x^2-1}}$	11. $f(x) = \frac{1}{\lvert x\rvert\sqrt{x^2-1}} \Rightarrow F(x) = \sec^{-1} x + C$
12. $(e^x)' = e^x$	12. $f(x) = e^x \Rightarrow F(x) = e^x + C$
13. $(\ln x)' = \frac{1}{x}$	13. $f(x) = \frac{1}{x} \Rightarrow F(x) = \ln x + C$
14. $(\sinh x)' = \cosh x$	14. $f(x) = \cosh x \Rightarrow F(x) = \sinh x + C$
15. $(\cosh x)' = \sinh x$	15. $f(x) = \sinh x \Rightarrow F(x) = \cosh x + C$
16. $(\tanh x)' = \operatorname{sech}^2 x$	16. $f(x) = \operatorname{sech}^2 x \Rightarrow F(x) = \tanh x + C$
17. $(\operatorname{sech} x)' = -\operatorname{sech} x \tanh x$	17. $f(x) = \operatorname{sech} x \tanh x \Rightarrow F(x) = -\operatorname{sech} x + C$
18. $(\operatorname{csch} x)' = -\operatorname{csch} x \coth x$	18. $f(x) = \operatorname{csch} x \coth x \Rightarrow F(x) = -\operatorname{csch} x + C$
19. $(\coth x)' = -\operatorname{csch}^2 x$	19. $f(x) = \operatorname{csch}^2 x \Rightarrow F(x) = -\coth x + C$

Find the general antiderivatives of the given functions:

1. $f(x) = 3x^2$

2. $f(x) = 10x^4 + 12x^3 - 30x^2 + 8x + 5$

3. $f(x)=x^4+6x^3+7x+3$

4. $f(x)=12\sqrt{x}$

5. $f(x)=\sin x+\cos x$

6. $f(x)=\sec^2 x+\sec x\tan x$

7. $f(x)=x\left(x^2+1\right)^2$

8. $f(x)=\frac{x^4+3x^2+1}{x^2}$

9. $f(x)=\frac{1}{x}+\frac{1}{1+x^2}+\sinh x+\cosh x+\operatorname{sech}^2 x+\operatorname{sech} x\tanh x+e^x$

10. Find $f(x)$ if $f'(x)=6x+5$ and $f(1)=10$

11. Find $f(x)$ if $f''(x)=12x+2$ and $f(1)=5$ and $f'(2)=31$

12. Find $f(x)$ if $f'(x)=10x^4+\sin x+\cos x+\sinh x+\cosh x+\frac{1}{\sqrt{1-x^2}}+e^x$ and $f(0)=4$