Worksheet #29: Sigma Notation, The Definite Integral as the Limit of a Sum

Useful Formulas: If $f(x)$ is continuous on $[a,b]$, then the

$Signed\ Area = \int_a^b f(x)dx = \lim_{n\to+\infty} \sum_{k=1}^{n} f(x_k)\Delta x$ (right endpoint, equal subintervals) $x_k = a + k\Delta x,\ \Delta x = \frac{b-a}{n}$.

$\sum_{k=1}^{n} 1 = n$ $\quad \sum_{k=1}^{n} k = \frac{n(n+1)}{2}$ $\quad \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$ $\quad \sum_{k=1}^{n} k^3 = \left[\frac{n(n+1)}{2}\right]^2$

1. Evaluate $\sum_{k=1}^{3}(2k^2 - k^3) + \sum_{k=1}^{10}(4k+1)$.

2. Find the area under the curve $f(x) = 2 + x^3$ over the interval $[a,b]$, where x_k is the right endpoint of each equal subinterval. Given $a = 0$ and $b = 2$.

3. Evaluate the definite integral as the limit of the sum: $\int_a^b f(x)dx = \lim_{n\to+\infty} \sum_{k=1}^{n} f(x_k)\Delta x$, evaluate $\int_1^4 (2x+3)dx$.