Worksheet #3: Continuity

Definition: *f is continuous at $x = c$ if ALL of the following conditions are true:*

(i) f(c) exists *(ii) $\lim_{x \to c} f(x)$ exists* *(iii) $\lim_{x \to c} f(x) = f(c)$.*

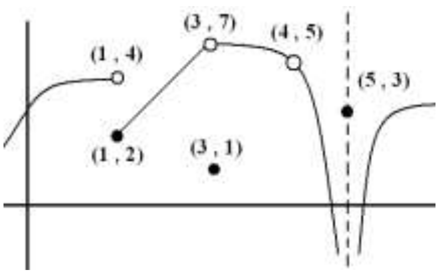
*A discontinuity is said to be **removable** when the graph has a "hole" that can be plugged by a single point. Otherwise, a discontinuity is said to be **nonremovable**.*

Theorem: *If the functions f and g are continuous at c then (a) $f(x) \pm g(x)$ is continuous at c, (b) $f(x)g(x)$ is continuous at c, (c) $f(x)/g(x)$ is continuous at c if $g(c) \neq 0$ or $f(x)/g(x)$ has a discontinuity at c if $g(c) = 0$.*

1. a) Give the three conditions that are necessary for the function *f(x)* to be continuous at $x = c$.

(i)
(ii)
(iii)

b) From the graph of $f(x)$ below, find the values of x where $f(x)$ is discontinuous and state all conditions in part (a) that fail to hold. State whether the discontinuities are *removable* or *nonremovable*.



2. Given $f(x) = \begin{cases} kx^2, x \leq 2 \\ 2x + k, x > 2 \end{cases}$.

Find a value for the constant *k* that will make the function *f(x)* continuous everywhere:

3. Find the values of x at which $f(x)$ is discontinuous. Use interval notation (if possible) to describe the set of values where $f(x)$ is continuous.

(a) $f(x) = \dfrac{e^x}{x-2}$, (b) $f(x) = x\ln(x)$, (c) $f(x) = \cot(x) = \dfrac{\cos(x)}{\sin(x)}$, (d) $f(x) = \dfrac{\sqrt{x-5}}{x^2 - 2x - 3}$, (e) $f(x) = x^4 - 3x^2 + 8$