Worksheet #31: The Fundamental Theorem of Calculus

The Fundamental Theorem of Calculus: Let $f(x)$ be continuous on an interval I with $a<b$ and a, b, x in I.

Part 1: $F(x)=\int_a^x f(t)dt$ is an antiderivative of $f(x)$, that is $F'(x)=\frac{d}{dx}\left(\int_a^x f(t)dt\right)=f(x)$.

Or more generally, combining with chain rule when integration limits are functions

$F(x)=\int_{v(x)}^{u(x)} f(t)dt$ is an antiderivative of $f(x)$, that is $F'(x)=\frac{d}{dx}\left(\int_v^u f(t)dt\right)=f(u)\frac{du}{dx}-f(v)\frac{dv}{dx}$.

Part 2: $\int_a^b f(x)dx=F(b)-F(a)$ for any $F(x)$ that is an antiderivative of $f(x)$.

Evaluate the following integrals by using the Fundamental Theorem of Calculus:

1. $\int_1^2 (3x^2-2x+1)dx=$

2. $\int_0^{\frac{\pi}{2}} \sin x dx=$

3. $\int_1^2 x(2-x)dx=$

4. $\int_0^{\frac{\pi}{3}} \sec x \tan x dx=$

5. $\int_{-1}^1 \sqrt[3]{x} dx=$

6. $\int_0^{\frac{\pi}{4}} (x-\sec x \tan x)dx=$

7. $\int_1^4 \frac{1}{\sqrt{x}} dx=$

8. $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \left(\frac{2}{\cos^2 x}\right) dx =$

Use Part 1 of the FTC to find:

9. $\frac{d}{dx}\left(\int_1^x \sec^2(t)\,dt\right) =$

10. $\frac{d}{dx}\left(\int_\pi^x \sin\left(\sqrt{t}\right) dt\right) =$

11. $\frac{d}{dx}\left(\int_4^{x^3} T^2\,dT\right) =$

12. $\frac{d}{dx}\left(\int_{3x}^{x^2} T^3\,dT\right) =$