Worksheet #32: Indefinite Integrals

Useful derivative formulas:

1. $\frac{d}{dx}[x]=1$
2. $\frac{d}{dx}\left[x^r\right]=rx^{r-1}$
3. $\frac{d}{dx}[\sin(x)]=\cos(x)$
4. $\frac{d}{dx}[\cos(x)]=-\sin(x)$
5. $\frac{d}{dx}[\tan(x)]=\sec^2(x)$
6. $\frac{d}{dx}[\cot(x)]=-\csc^2(u)$
7. $\frac{d}{dx}[\sec(x)]=\sec(x)\tan(x)$
8. $\frac{d}{dx}[\csc(x)]=-\csc(x)\cot(x)$
9. $\frac{d}{dx}[\tan^{-1}(x)]=\frac{1}{1+x^2}$
10. $\frac{d}{dx}[\sin^{-1}(x)]=\frac{1}{\sqrt{1-x^2}}$
11. $\frac{d}{dx}[\sec^{-1}(x)]=\frac{1}{x\sqrt{x^2-1}}$
12. $\frac{d}{dx}[\ln|x|]=\frac{1}{x}$
13. $\frac{d}{dx}[e^x]=e^x$
14. $\frac{d}{dx}[b^x]=b^x\ln b$
15. $\frac{d}{dx}[\sinh(x)]=\cosh(x)$
16. $\frac{d}{dx}[\cosh(x)]=\sinh(x)$

Produce the table of indefinite integrals that correspond to the above derivatives [Three are given].

1) $\int dx=x+C$
2) $\int x^t dx=$
3) $\int \cos(x)dx=$
4) $\int \sin(x)dx=$
5) $\int \sec^2(x)dx=$
6) $\int \csc^2(x)dx=-\cot(x)+C$
7) $\int \sec(x)\tan(x)dx=$
8) $\int \csc(x)\cot(x)dx=$
9) $\int \frac{1}{1+x^2}dx=$
10) $\int \frac{1}{\sqrt{1-x^2}}dx=$
11) $\int \frac{1}{x\sqrt{x^2-1}}dx=\sec^{-1}x+C$
12) $\int \frac{1}{x}dx=$
13) $\int e^x dx=$
14) $\int b^x dx=$
15) $\int \sinh(x)dx=$
16) $\int \cosh(x)dx=$

Evaluate the following indefinite integrals using the table of integrals you produced:

1. $\int 70x^9 dx =$

2. $\int \left(20x^4 + 8x^3 - 24x^2 + 14x + 9\right) dx =$

3. $\int (x^4 + 10x^3 + 3x + 5) dx =$

4. $\int 60\sqrt[3]{x^2}\, dx =$

5. $\int \left(3\cos x + 4\sin x\right) dx =$

6. $\int \sec x(\sec x + \tan x) dx =$

7. $\int x^3 + x^{-3} dx =$

8. $\int \frac{\cos x}{\sin^2 x} dx =$

9. $\int x\left(x^3 + 5\right)^2 dx =$

10. $\int \left(\frac{x^4 + \sqrt{x}}{x^2}\right) dx =$

11. $\int \sqrt{\frac{2}{x}} dx =$

12. $\int \left(e^x + \frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^2+1}\right) dx =$

5.4 Net Change

The indefinite integral of a derivative recovers the original function (plus a constant of integration)

$$\int F'(x)dx = F(x) + C$$

The definite integral of a rate of change is the net change

$$\int_a^b F'(x)dx = F(b) - F(a)$$

The total distance traveled may differ from the net change in position (called displacement)

Total distance= $\int_a^b |s'(x)|\,dx$ Displacement= $\int_a^b s'(x)\,dx$

13. If the acceleration equation of an object is $a(t) = 6t + 12$, fine the velocity function $v(t)$ and the position function $s(t)$ if $v(1) = 5$ and $s(2) = 4$.

14. A toy car moves along a straight track during time $0 \le t \le 3$. It's velocity at any time is given by $v(t) = 6t - 6$. Find the car's displacement, or net change in position. Is this the same as the total distance traveled by the car? It if differs from displacement, calculate the total distance traveled.

Total distance differs because of a sign change in *v* at *t=1*.