Worksheet #33: The Substitution Rule

Sometimes we may reduce a complicated integral into a simpler one by substitution. To do so, choose some part of the integrand to be re-labeled as a new variable *u*. Then proceed to rewrite the integral in terms of *u* and the *dx*-term in terms of *du*. If you are successful, a simpler integral will result:

$$\int f(x)dx \Rightarrow \text{Let } u = g(x) \Rightarrow \tfrac{du}{dx} = g'(x) \Rightarrow du = g'(x)\,dx \Rightarrow dx = \tfrac{du}{g'(x)}$$

Relabeling: $$\underset{\text{Old } x\text{ - Funct} \Rightarrow \text{New } u\text{ - Function}}{\int f(x)dx = \int h(u)du}$$

The process of Integration by *u*-substitution is identical to the process for indefinite integrals *except* that when there are limits involved in the definite integral, the *x*-limits in the original problem must be converted to new *u*-limits before they can be used and plugged in:

If $\int_{x=a}^{x=b} f(x)dx$ and you make the substitution $u = g(x)$ then $\begin{matrix} x = b \Rightarrow u = g(b) \\ x = a \Rightarrow u = g(a) \end{matrix}$

and so $\int_{x=a}^{x=b} f(x)dx = \int_{u=g(a)}^{u=g(b)} h(u)du$

Evaluate the following integrals:

1. $\int \left(2x\left(x^2+1\right)^{23}\right)dx$ $\quad u =$ $\quad du =$

2. $\int \left(2x^3-2x\right)^5\left(3x^2-1\right)dx$ $\quad u =$ $\quad du =$

3. $\int \left(\cos^3(x)\sin(x)\right)dx$ $\quad u =$ $\quad du =$

4. $\int \sin(3x)dx$ $\quad u =$ $\quad du =$

5. $\int x(x-1)^5 dx$ $u =$ $du =$

6. $\int_{x=0}^{x=1} \left[80x(x^2+1)^3\right] dx$ $u =$ $du =$

7. $\int_{x=0}^{x=\frac{\pi}{4}} \left[16\cos^3(x)\sin(x)\right] dx$ $u =$ $du =$

8. $\int_{x=0}^{x=4} \sqrt{2x+1}\, dx$ $u =$ $du =$

9. $\int_{x=-1}^{x=0} \frac{2}{(1-2x)^2} dx$ $u =$ $du =$