Worksheet #34: Sequences

Basic Notions:

* A sequence is a function whose domain is a set of integers. $f(n)=a_n,\ \ n=1,2,3,\ldots.$
* A sequence $\{a_n\}_{n=1}^{+\infty}$ converges to L (finite), if $\lim_{n\to\infty} a_n = L.$
* A sequence $\{a_n\}_{n=1}^{+\infty}$ has at most a single limit L (finite). If the terms a_n are not getting closer to a single finite value, we say that the sequence diverges.
* *If* $x\in[1,\infty),$ *then* $\lim_{x\to\infty} f(x)=L \ \Rightarrow \lim_{n\to\infty} f(n)=\lim_{n\to\infty} a_n = L.$
* A sequence is *increasing* if its terms are getting larger: $a_n < a_{n+1}$
* A sequence is *decreasing* if its terms are getting smaller: $a_n > a_{n+1}$

There are three simple ways to check whether the given sequence $\{a_n\}$ is increasing or decreasing:

1. Subtraction Method: If $a_{n+1}-a_n>0$, then the sequence is increasing
   If $a_{n+1}-a_n<0$, then the sequence is decreasing
2. Division Method: If $a_{n+1}/a_n>1$, then the sequence is increasing
   If $a_{n+1}/a_n<1$, then the sequence is decreasing
3. Derivative Method: Replace the *n*'s in the series with *x*'s to create a similar function $f(x)$ and look at $f'(x)$
   If $f'(x)>0$ for $x\ge 1$, then the sequence is increasing
   If $f'(x)<0$ for $x\ge 1$, then the sequence is decreasing.

Write out the first four terms of the sequence, try to find its limit, and determine whether the sequence converges or diverges.

1. (a) $\left\{\dfrac{\ln n}{5n}\right\}_{n=1}^{+\infty}$ (b) $\{7\}_{n=1}^{+\infty}$

2. (a) $\left\{(-1)^n\dfrac{4n^3}{n^3+2}\right\}_{n=1}^{+\infty}$ (b) $\left\{(-1)^{n+1}\dfrac{1}{n^3}\right\}_{n=1}^{+\infty}$

3. (a) $\left\{\sqrt{n^2+2n}-n\right\}_{n=1}^{+\infty}$ (b) $\left\{(1-\frac{1}{n})^n\right\}_{n=1}^{+\infty}$

4. Use the Subtraction Method to show that the given sequence is increasing or decreasing.

(a) $\left\{4-\frac{1}{n}\right\}_{n=1}^{+\infty}$ (b) $\left\{7^n-n\right\}_{n=1}^{+\infty}$

5. Use the Division Method to show that the given sequence is increasing or decreasing.

(a) $\left\{\frac{2n}{n+1}\right\}_{n=1}^{+\infty}$ (b) $\left\{\frac{9^n}{(2n)!}\right\}_{n=1}^{+\infty}$

6. Use the Derivative Method to show that the given sequence is increasing or decreasing.

(a) $\left\{ne^{-3n}\right\}_{n=1}^{+\infty}$ (b) $\left\{\frac{1}{3+\ln(n)+n^2}\right\}_{n=1}^{+\infty}$