Worksheet #37: Infinite Series

Basic Guidelines:

The sequence of partial sums is $\{s_n\}_{n=1}^{\infty}$ where $s_n = \sum_{k=1}^{n} u_k$.

If the $\lim_{n\to+\infty} s_n = S(\textit{finite})$, then S is the sum of the infinite series and $S = \lim_{n\to+\infty} \sum_{k=1}^{n} u_k = \sum_{k=1}^{\infty} u_k$.

A geometric series $\sum_{k=0}^{+\infty} ar^k = a + ar + ar^2 + \ldots + ar^k + \ldots (a \neq 0)$ converges if $|r| < 1$ and diverges if $|r| \geq 1$.

If the geometric series converges, then the sum S is the $\lim_{n\to+\infty} s_n = \lim_{n\to+\infty} \sum_{k=0}^{n} ar^k = \sum_{k=0}^{\infty} ar^k = \frac{a}{1-r}$.

Divergence Test: In order for a series $\sum_{n=0}^{\infty} a_n$ to converge, the terms a_n must be going to zero. Just because the terms in the series are going to zero does not automatically mean that it converges. However, if they go to any other limit besides zero, the series definitely *diverges*: If $\sum_{n=0}^{\infty} a_n$ and $\lim_{n\to\infty} a_n = L \neq 0$, then $\sum_{n=0}^{\infty} a_n$ diverges.

Determine whether the series converge or diverge. If they converge, evaluate the sum.

1. (a) $\sum_{k=0}^{+\infty} \left(\frac{-1}{2}\right)^{k+1}$ (b) $\sum_{k=0}^{+\infty} 2^{2k} 3^{1-k}$

2. (a) $\sum_{k=0}^{+\infty} \left(\frac{e}{3}\right)^{k-1}$ (b) $\sum_{k=1}^{+\infty} (-1)^{k-1} \frac{4}{3^{k-1}}$

3. Use a PFD to construct a telescoping series. $\sum_{k=1}^{+\infty} \frac{1}{k^2 + 5k + 6}$

4. Use the Divergence Test to determine what, if anything, can be said about the convergence of the following series:

(a) $\sum_{k=1}^{+\infty} \frac{k^3}{k^3 + 7}$ (b) $\sum_{k=1}^{+\infty} \frac{9k}{e^k}$