

Worksheet #4: Computing Limits at Infinity, End Behavior

End behavior: $r > 0$

$$\lim_{x \rightarrow +\infty} \frac{1}{x^r} = 0, \lim_{x \rightarrow -\infty} \frac{1}{x^r} = 0, \quad \lim_{x \rightarrow +\infty} e^x = +\infty, \lim_{x \rightarrow -\infty} e^x = 0 \quad \lim_{x \rightarrow +\infty} \tan^{-1}(x) = \frac{\pi}{2}, \lim_{x \rightarrow -\infty} \tan^{-1}(x) = -\frac{\pi}{2}$$

Some helpful hints in computing the limits:

- 1: Try to reason out what happens as $x \rightarrow +\infty$ or $-\infty$, which terms dominate? Which terms go to zero?
- 2: For an $\frac{\infty}{\infty}$ form, try to simplify using algebra: divide by the highest power in the denominator (multiply by 1) or factor out the highest power, etc.
- 3: For $\infty - \infty$ form, use algebra to simplify: multiply by 1 using the conjugate.

Find the limit for the following:

1. $\lim_{x \rightarrow +\infty} \frac{9x^2 - 4x}{3x + 1}$

2. $\lim_{x \rightarrow +\infty} \sqrt[3]{\frac{7 - 3x + 5x^2}{3 + 7x^2}}$

3. $\lim_{x \rightarrow +\infty} \frac{3x^2 + 7x + 2}{x^2 + 4x + 4}$

4. $\lim_{x \rightarrow -\infty} \frac{12x + 7}{\sqrt{36x^2 - 15}}$

5. $\lim_{x \rightarrow +\infty} x^2 - \sqrt{x^4 + 10}$

6. $\lim_{x \rightarrow +\infty} (3 + e^{-x^2})$

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8. $\lim_{x \rightarrow +\infty} \tan^{-1}(\pi - x^3)$