Worksheet #40: Integral Test for Conversion of an Infinite Series

Basic Guidelines:

1. Convergence of p-series: $\sum_{k=1}^{+\infty} \frac{1}{k^p}$ is a p-series and converges if $p > 1$ and diverges if $0 < p \le 1$.

3. The Integral Test: Let $\sum_{k=1}^{+\infty} u_k$ be a series with positive terms, and let *f(x)* be the function that results when *k* is replaced by *x*. If *f* is decreasing and continuous on the interval $[1, +\infty)$, then $\sum_{k=1}^{+\infty} u_k$ and $\int_1^{+\infty} f(x)dx$ both converge or both diverge.

1. Use the Divergence Test to reach a conclusion about the following series. If the Divergence test fails to give any useful information, try using the Integral Test to learn more:

   (a) $\sum_{k=1}^{+\infty} \frac{k^3}{k^3 + 7}$ (b) $\sum_{k=1}^{+\infty} \frac{9k}{e^k}$

2. Find p and determine whether the p-series converges or diverges.

   (a) $\sum_{k=1}^{+\infty} \frac{1}{\sqrt[5]{k}}$ (b) $\sum_{k=1}^{+\infty} k^{-e}$

3. Check that the integral test is applicable, and then use it to determine whether the series converges.

   (a) $\sum_{k=5}^{+\infty} \frac{2k}{8 + k^2}$ (b) $\sum_{k=1}^{+\infty} \frac{1}{4 + k^2}$