

## Worksheet #41 The Comparison Test

### Basic Guidelines:

Let  $\sum_{n=1}^{\infty} a_n$  and  $\sum_{n=1}^{\infty} b_n$  be series with positive terms.

(1) Direct Comparison

If  $a_n \leq b_n$  and  $\sum_{n=1}^{\infty} b_n$  converges, then  $\sum_{n=1}^{\infty} a_n$  converges.

If  $b_n \leq a_n$  and  $\sum_{n=1}^{\infty} b_n$  diverges, then  $\sum_{n=1}^{\infty} a_n$  diverges.

(2) The Limit Comparison Test: If  $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = L$  and  $L > 0$  and finite, then both  $\sum_{n=1}^{\infty} a_n$  and  $\sum_{n=1}^{\infty} b_n$  behave the same. That is, both series converge or both diverge.

Use a direct comparison to determine whether the series converges or diverges.

1.  $\sum_{n=1}^{\infty} \frac{(\cos n)^2}{n^3}$

2.  $\sum_{n=1}^{\infty} \frac{2n}{3n^2 - 1}$

Use the limit comparison test to determine whether the series converges or diverges.

3.  $\sum_{n=1}^{\infty} \frac{1}{3n+5}$

4.  $\sum_{n=1}^{\infty} \frac{3n^2+5}{7n^6}$

5.  $\sum_{n=1}^{\infty} \frac{1}{3^n - 1}$

6.  $\sum_{n=1}^{\infty} \frac{3n^2+1}{\sqrt{5n^6-2}}$