Worksheet #43: Absolute & Conditional Convergence, The Ratio & Root Tests for Absolute Convergence

Basic Guidelines:

1. The series $\sum_{n=1}^{\infty} u_n$ *converges absolutely* if $\sum_{n=1}^{\infty} |u_n|$ converges.
2. The series $\sum_{n=1}^{\infty} u_n$ *converges conditionally* if $\sum_{n=1}^{\infty} |u_n|$ diverges and $\sum_{n=1}^{\infty} u_n$ converges.
3. The Ratio Test for Absolute Convergence:

   If $\lim_{n\to\infty} \frac{|u_{n+1}|}{|u_n|} = L$ and $L < 1$, then the series $\sum_{n=1}^{\infty} u_n$ converges absolutely. If $L > 1$, then the series diverges.

   If $L = 1$, then the test provides no useful information and another test must be used.
4. The Root Test for Absolute Convergence:

   If $\lim_{n\to\infty} \sqrt[n]{|u_n|} = L$ and $L < 1$, then the series $\sum_{n=1}^{\infty} u_n$ converges absolutely. If $L > 1$, then the series diverges.

   If $L = 1$, then the test provides no useful information and another test must be used.

Determine whether the following series converge absolutely, converge conditionally or diverge.

1. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{\sqrt{n+4}}$

2. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{10^n}{n!}$

3. $\sum_{n=3}^{\infty} (-1)^{n+1} \frac{1}{n\ln(n)}$

4. $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{k+1}{k^3+k}$

5. $\sum_{n=0}^{\infty} (-1)^n \left(\frac{5n^2-7}{3n^2+1}\right)^n$