Worksheet #45: Parametric Equations II

Parametric equations have their x and y-coordinates defined in terms of another parameter: $x = f(t)$ & $y = g(t)$
Familiar formulas may be written parametrically, such as

First derivative (tangent line slope): $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{g'(t)}{f'(t)}$

Second derivative (concavity): $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d\left(\frac{dy}{dx}\right)}{dx} = \frac{d\left(\frac{dy}{dx}\right)/dt}{dx/dt} = \frac{\left(\frac{g'(t)}{f'(t)}\right)'}{f'(t)} = \frac{\frac{g''(t)f'(t)-f''(t)g'(t)}{[f'(t)]^2}}{f'(t)} = \frac{g''(t)f'(t)-f''(t)g'(t)}{[f'(t)]^3}$

Arc Length: $L = \int_{t=a}^{b} ds = \int_{t=a}^{b} \sqrt{dx^2 + dy^2} = \int_{a}^{b} \sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2}\, dt = \int_{a}^{b} \sqrt{[f'(t)]^2 + [g'(t)]^2}\, dt$

Area of a surface of revolution about the x-axis: $S.A. = \int_{t=a}^{b} 2\pi[g(t)]\sqrt{[f'(t)]^2 + [g'(t)]^2}\, dt$

Area of a surface of revolution about the y-axis: $S.A. = \int_{t=a}^{b} 2\pi[f(t)]\sqrt{[f'(t)]^2 + [g'(t)]^2}\, dt$

1. Find expressions for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ in terms of t for the following parametric equations:

   $x = t^2$
   $y = t^3$

2. Find the arc length of the parametric curve for $x = \cos(3t)$, $y = \sin(3t)$, where $0 \le t \le \frac{\pi}{9}$.

3. Set-up and evaluate the definite integral that represents the area of the surface generated by revolving the given curve about the y-axis:

   $x = 3t$ and $y = 4t$ with $0 \le t \le 2$