Worksheet #5: Rates of Change

A "Rate of Change" of a function f(x) over a given x-interval is found by doing a slope calculation.
Average Rates of Change *involve two points* $x=a$ *and* $x=b$ *and can be calculated using the traditional two-point slope formula:* $m=\dfrac{f(b)-f(a)}{b-a}$

Instantaneous Rates of Change *involve only one point* $x=a$ *and must be calculated using a limit:*

$$m=\lim_{h\to 0}\frac{f(a+h)-f(a)}{h} \quad \text{or} \quad m=\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$$

1. Given the function $f(x)=2x^2+4x+5$, find its *average rate of change* between the points $x=1$ and $x=5$.

2. Given the function $f(x)=2x^2+3x+1$, find its *instantaneous rate of change* at the point $x=1$ by two different methods:

   a. Calculate the *instantaneous rate of change* using the formula: $m=\lim\limits_{h\to 0}\dfrac{f(a+h)-f(a)}{h}$.

   b. Calculate the *instantaneous rate of change* using the formula: $m=\lim\limits_{x\to a}\dfrac{f(x)-f(a)}{x-a}$.

3. Given the function $f(x)=x^3$, find its *instantaneous rate of change* at the point $x=2$ by two different methods:

   a. Calculate the *instantaneous rate of change* using the formula: $m=\lim\limits_{h\to 0}\dfrac{f(a+h)-f(a)}{h}$.

   b. Calculate the *instantaneous rate of change* using the formula: $m=\lim\limits_{x\to a}\dfrac{f(x)-f(a)}{x-a}$.