

Worksheet #6: Definition of the Derivative

Definition of a Derivative: let $y = f(x)$ be a differentiable function, then the derivative is defined by

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \quad \text{or} \quad f'(x) = \lim_{w \rightarrow x} \frac{f(x) - f(w)}{x - w}$$

1. Given $f(x) = 2x^2$.

a. Find $f'(x)$ by using the definition of derivative $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$.

b. Find the equation of the tangent line to the graph of $y = f(x)$ at $x = 3$ in the “ $y = mx + b$ ” form.

2. Find $\frac{dy}{dx}$ by using the definition of a derivative where $y = \frac{1}{x+1}$, then find the equation of the tangent line at $x = 1$.

3. Let $f(x) = \sqrt{x}$. Using only the definition of a derivative, evaluate $\left. \frac{dy}{dx} \right|_{x=100}$.