

Worksheet #8: The Product and Quotient Rules

Useful Formulas:

$$1. \frac{d}{dx}[x^n] = nx^{n-1} \qquad 2. \frac{d}{dx}[c] = 0 \qquad 3. \frac{d}{dx}[cf(x)] = c \frac{d}{dx}[f(x)] = cf'(x)$$

$$4. \frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}[f(x)] \pm \frac{d}{dx}[g(x)] = f'(x) \pm g'(x) \qquad 5. \frac{d}{dx}[e^x] = e^x$$

$$6. \frac{d^2}{dx^2}(y) = \frac{d^2y}{dx^2} = f^{(2)}(x) = f''(x) = (f'(x))'$$

$$7. \frac{d^n}{dx^n}(y) = \frac{d^ny}{dx^n} = f^{(n)}(x)$$

Product Rule: $\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)] = fg' + gf'$; $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx} = uv' + vu'$

Quotient Rule: $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\frac{d}{dx}[f(x)] - f(x)\frac{d}{dx}[g(x)]}{[g(x)]^2} = \frac{gf' - fg'}{g^2}$

For the following problems, you need not simplify your answers.

1. Find $f'(x)$ for the following:

(a) $f(x) = (2x^3 + 9x)(4x + 1)$

(b) $f(x) = (x^3 + 7x^2 - 2x)(3x^7 + 8x - 1)$

2. Find $\frac{dy}{dx}$ for $y = \left(9x - \frac{2}{x}\right)(3x^9 + 6\sqrt{x})$ and $\left.\frac{dy}{dx}\right|_{x=1}$.

3. Find $\frac{dy}{dx}$ for $y = \frac{12x^3 + 3x}{7x^2 + \frac{1}{x}}$.

4. Find $\frac{dy}{dx}$ for $y = \frac{10\sqrt{x} - 4x}{6x^3 + 3x}$.

5. Find the equation of the tangent line to the curve $y = \frac{e^x}{x+1}$ at the point $x = 0$.

6. Find $f''(x)$ for $f(x) = 2x^3e^x$, this time simplify your answer.