Worksheet #9: Derivatives of the Trigonometric Functions

Useful Formulas:

Limit Formulas: $\lim_{\theta\to 0}\frac{\sin(\theta)}{\theta}=1$ $\lim_{\theta\to 0}\frac{\cos(\theta)-1}{\theta}=0$

Derivative Formulas:

$\frac{d}{dx}[\sin(x)]=\cos(x)$ $\frac{d}{dx}[\cos(x)]=-\sin(x)$

$\frac{d}{dx}[\tan(x)]=\sec^2(x)$ $\frac{d}{dx}[\cot(x)]=-\csc^2(x)$

$\frac{d}{dx}[\sec(x)]=\sec(x)\tan(x)$ $\frac{d}{dx}[\csc(x)]=-\csc(x)\cot(x)$

1. If $f(x)=\frac{\sin(2x)}{x}$, use a graphing calculator to evaluate the following to seven decimal places:
[Remember, the values of the *x*-coordinates are in *radians*!]

$f(.1)=$ $f(.01)=$ $f(.001)=$

Use your above answers to make a conjecture about: $\lim_{x\to 0^+}\frac{\sin(2x)}{x}=$

Use a graphing calculator to graph the function $f(x)=\frac{\sin(2x)}{x}$ and confirm your answer.
Draw the graph below:

2. Find $\frac{dy}{dx}$ for the following: a. $y=\sec(x)\tan(x)$ b. $y=4\csc(x)\cot(x)$

3. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for the following: a. $y=\sec(x)$ b. $y=\tan(x)$

4. Find $f'(x)$ and $f'(\pi/2)$ for $f(x) = (4x^2 + 4x)(\cot(x))$.

5. Find $\frac{d}{dx}[\frac{12x^3 \csc(x)}{3\sin(x) + 3x^4}]$.

6. Use the definition of a derivative $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ to find the derivative of $f(x) = \cos(x)$.

   Hint: Recall that $\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$

7. Use the known derivatives for sin *x* and cos *x* to derive the formula for the derivative of csc *x* by using the Quotient Rule and known trig identities.

$\csc x =$

$\left(\csc x\right)' =$